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Drifter Motion Planning for Optimal Surveillance of the Ocean
Progress Report

Stochastic Prediction and Control in Multiscale Systems

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PROBLEM AND APPROACH

The current problems of the proposal deal with littoral monitoring of large regions of the ocean using single or multiple gliders. This portion of the joint proposal with UCSB deals with the stochastic modeling and control of gliders in the ocean environment. Basically, the goals are to model stochastic flows on gliders, considered as particles forced by the ocean, and design optimal control to keep the gliders in a specific region. That is, the control will be designed to increase loitering time in a specified region. Without controls, the stochastic forcing of the environment will cause the gliders to leave the region of interest in finite time. Controls were designed to optimize the lifetime by minimizing the actuation intervals.

RESEARCH TASKS AND OVERALL PROGRESS

a. Here we designed a model of a glider on the ocean surface which is acted upon by stochastic forcing. We modeled the glider as a point particle, and considered a multiscale flow in time. Based on the model of the flow, we are able to analytically predict the probability of escape, and the mean lifetimes in a finite region..

b. New mathematical theories of stochastic model reduction were developed to analyze the probability of escape. Using these new tools, an optimal path to escape can be identified. Using this path, optimal stochastic controls were derived to extend the lifetime of the specified region, thereby enhancing loitering times.

c. Multi-particle in stochastic environments: We considered the effect of communication delay, or latency, between multiple sensing platforms in stochastic flows. Here we showed how latency may destabilize the sensing swarm so that a particular patterns used for monitoring a given area are no longer stable.

d. Using networked local measurements to achieve global measurements; Here we used multiple sensors with communication to determine the large region surface temperature of the ocean.

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NONLINEAR STOCHASTIC MODEL-MULTIPLE SCALES IN A SINGLE LAYER OCEAN MODEL

We developed a model of the wind forced ocean in a single layer. The model takes into account multiple scales and stochastic effects. The model for vorticity is given by

$$\frac{\partial q}{\partial t} + J(\Psi, q) + \delta_S \nabla^2 \Psi + \delta_M \nabla^4 \Psi = A \sin\left(2\pi y + f(t)\right) \quad (1)$$

where the drive f(t) consists of a periodic component and additive stochastic terms. The drive also introduces multiple scales. The results of numerically approximating such a scheme are given as the velocity and stream function for generic parameters:

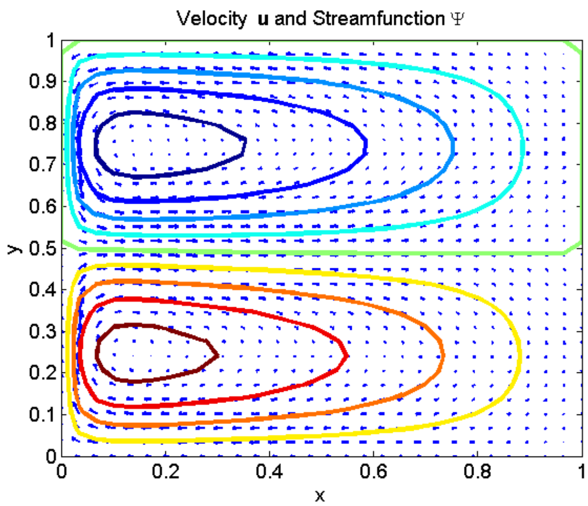

Fig. 1 Numerical realization of the stream function given in Eq. 1. The local velocity field is depicted by arrows. The two distinct regions are counter-rotating.

STOCHASTIC MODEL REDUCTION

Due to the high dimensionality and multiple scales coupled with the stochastic forcing, we derived a new method that projects the model onto a lower dimensional stochastic model. The basic problem of model reduction in deterministic models has been solved using center manifold theory. However, this will NOT work when stochastic models are included. Our technique is a novel mathematical construction that correctly captures the noise in the lower dimensional model. (E. Forgoston and I.B. Schwartz, Escape Rates in a Stochastic Environment with Multiple Scales, arXiv:0809.1345 in press SIAM J. Appl. Dyn. Systems (2009)).

We applied the technique to the simplest following problem:

$$\dot{x} = y + \phi_1,$$
$$\varepsilon \dot{y} = F(x, y) = \left(x - x^3 - y\right) + \phi_2 \qquad 2.$$

Here ε is a small parameter that is essentially the ration of the slow to fast time scales of the problem. The terms ϕ_{I}, I=1,2, are stochastic terms, which are Gaussian with intensity D. For small ε, the dynamics is almost one-dimensional. The results are shown in Fig. 2, which compares the theory and simulations for different noise intensities, D.

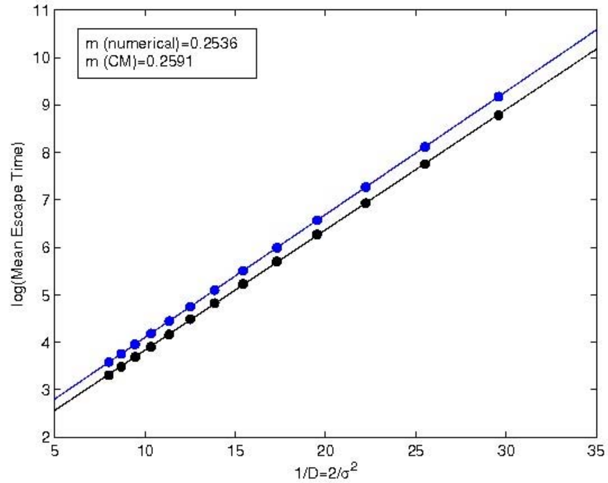

Fig. 2. Theory (blue) and simulations (black) of Eq. 2. as a function of noise intensity. The y-axis is the mean escape time from a specified region. There is no control implemented here. The stochastic manifold reduction was used to create a one-dimensional mode (theory curve), and the simulation data was for the full system. The exponents of the escape time are given by the slopes of the curves, and they are almost identical.

The correct noise projection using stochastic center manifold theory predicts maximal probability regions of escape, shown in Fig. 3, whereas the usual techniques for model reduction generate wrong statistics.

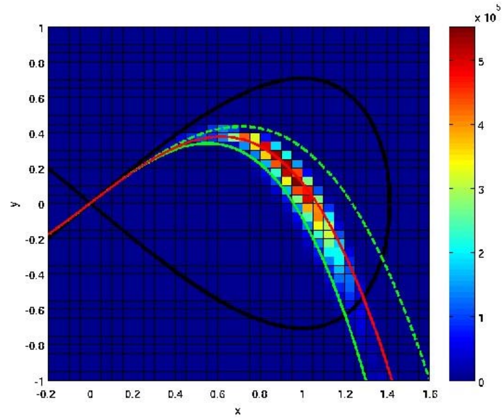

Fig. 3. Transition probability of escape is shown as the color bar. Higher probabilities are in red. The optimal path of escape is along the stochastic center manifold depicted by the red curve. (See arXiv:0809.1345 SIAM J. Appl. Dyn. Systems (2009) for details.).

TRANSPORT AND UNCERTAINTY FOR STOCHASTIC CONTROL

For the transport model in the single layer ocean model, we can sample the dynamics discretely every period. If the noise added is a random variable, η chosen from a distribution, ν, we have the map $F: M \rightarrow M, x \rightarrow F(x)$. The density evolution, ρ, may be computed using the Stochastic Frobenius-Perron (SFP)operator (Billings, L. and Schwartz ,IB. Identifying almost invariant sets in stochastic dynamical systems, CHAOS 18, art. No 023122 2008.).

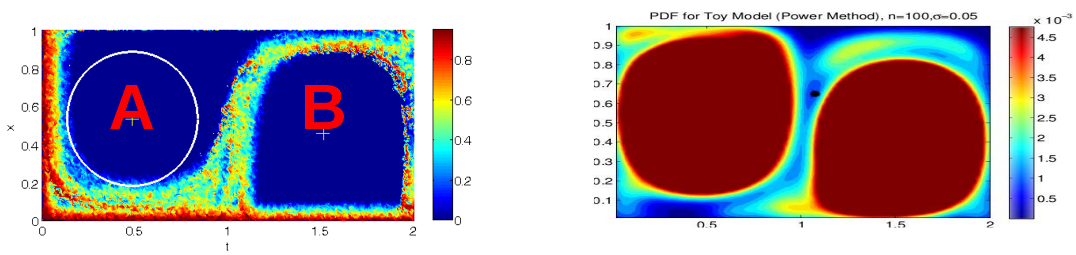

Fig. 4 (Left panel) Uncertainty regions are represented by the histogram colors, red being the most uncertain points. Uncertain points are those points which have a non-zero probability of escaping from A to B or vice versa. (Right panel) The brown regions are almost invariant sets which are computed using the SFP described above in the text. The colored regions have high transition probabilities going from one basin to the next.

The transport from one region to the next allows one to design effective monitoring regions for control. For example, if we wish to maintain a glider in region A of Fig. 4, then we just need to monitor the uncertain regions to maintain the dynamics in region A. This is the basic design of the control scheme. When coupled with the stochastic cener manifold approaches, it works exceedingly well, as in Fig.5 below.

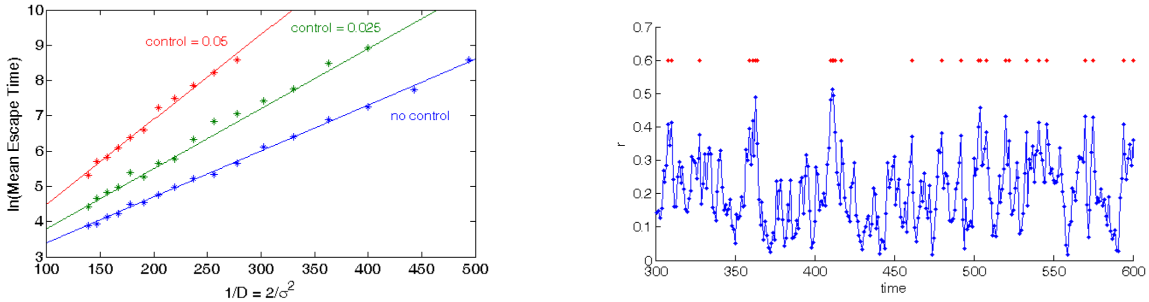

Fig. 5 (Left panel) Increase in loitering time with stochastic control for single layer stream model. Plotted is the mean escape time as a function of noise: no control (blue), control amplitude 0.025 (green), control actuation 0.06 (red). Notice the increase in slopes with greater control authority. (Right panel) A sample of a controlled time series. The control actuations are in red, and less than 5 per cent of the total loitering time.

Figure 5 shows that with even modest controls that are infrequent, we can achieve **exponentially** longer loitering times provided we can specify the uncertainty regions. Because our method is general, uncertainty specification may be done with real ocean models in the future, when coupled with data.

MULTI-PARTICLE SYSTEMS EFFECTS OF NOISE AND DELAY

We considered model of self-propelling particles interacting through pairwise forces in presence of noise and time delay. We found that time delay in communication between agents induces a transition that depends on the size of the coupling amplitude, and the transition is associated with a derived Hopf bifurcation. Analytical and numerical value of transitions agree very well, and can be seen in Forgoston, E. and Schwartz IB, Delay-induced instabilities in self-propelling swarms, Phys. Rev. E Rapid. Comm 77 035203 (2008)

ISSUES AND PLANS

We wish to continue the work, and unify the research done between NRL and UCSB, CUNY. In addition, some of the control work on the single layer ocean model has been done in collaboration with Montclair State (profs. Lora Billings and Phil Yecko), and they should be involved formally as well. There exist several issues which need to be considered in order to push the optimal control of gliders in the realm of 3D ocean modeling, and they are itemized below. For each issue, the plans subsection considers a viable approach to complete the project.

Issues

1.Improve stochastic control of standard stream models
2.Understanding of more realistic (harder) models
  - Decomposition into coherent structures and projected models-Intermediate goal
  - Analytic stochastic center manifold of 3D PDE ocean models-Long term goal
3.Incorporating real data and analysis of flow methods
4.Designs towards real time control of real 3D gliders.

Plans:

1.Incorporate optimal and stochastic controls-UCSB,CUNY, NRL
2.Use 3D layer ocean model (Montclair)

3.Use real ocean model (NAVCOM) using data assimilation to generate dynamic forces on a 3D glider.

- •Implement 3D glider model (We now have one.)
- •Using data/models, use prehistory and FTLE to describe dynamical regions and escape probabilities.

4.Implement stochastic optimal control scheme in 3D models.

STATISTICAL DATA

Transitions

None

Related Projects

Design of Cooperative Mobile Agents in collaboration with Northwestern university Department of Mechanical Engineering,

Publications

1. Forgoston, E. and Schwartz IB, Delay-induced instabilities in self-propelling swarms, Phys. Rev. E Rapid. Comm 77 035203 (2008)

2. Lynch, K. Schwartz IB et al, Decentralized environmental modeling by mobile sensor networks, IEEE Trans. Robotics 24, 710-724 (2008)

3. E. Forgoston and I.B. Schwartz, Escape Rates in a Stochastic Environment with Multiple Scales, arXiv:0809.1345, in press SIAM J. Appl. Dyn. Systems (2009).

Billings, L. and Schwartz ,IB. Identifying almost invariant sets in stochastic dynamical systems, CHAOS 18, art. No 023122 2008.

Patents

None

Honors/Awards/Prizes

Forgoston, E. and Schwartz IB, Delay-induced instabilities in self-propelling swarms, Phys. Rev. E Rapid. Comm 77 035203 (2008)

Alan Berman NRC/ASEE Best Paper award, 2008.